# Light Scattering by Small Particles

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# Outline

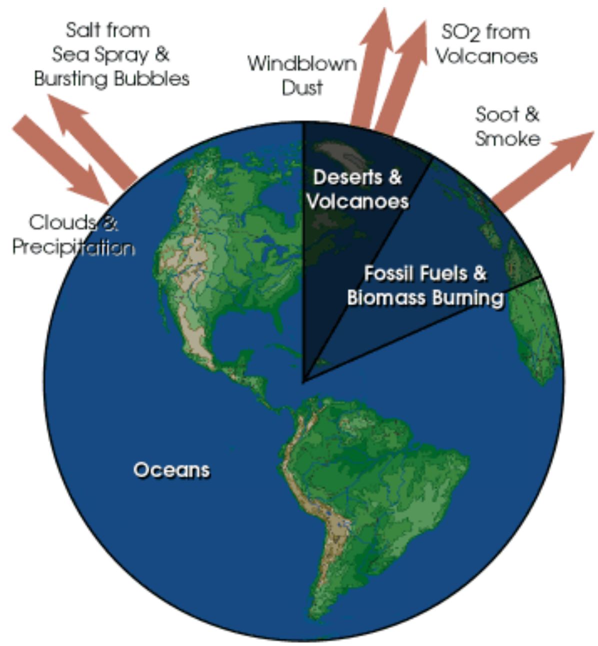
- Background introduction
- Theory
- Application
- Reference

Source: myskymom.com

*Have you seen any light scattering phenomena daily ?*

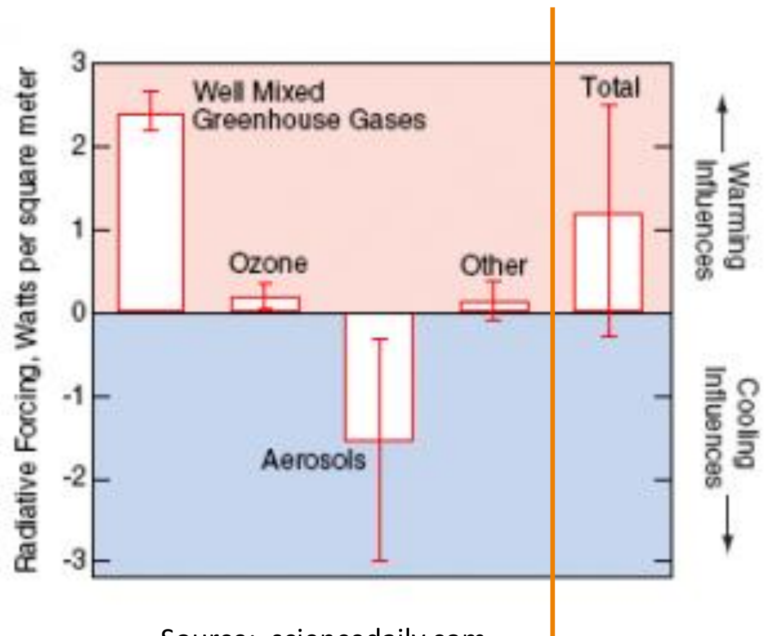
# Aerosol and Climate Change

- Aerosol: suspended particles in atmosphere
- Size: typical range 0.01~100um
- Source: nature (90%) and human activities (10%)




Source: nasa.org

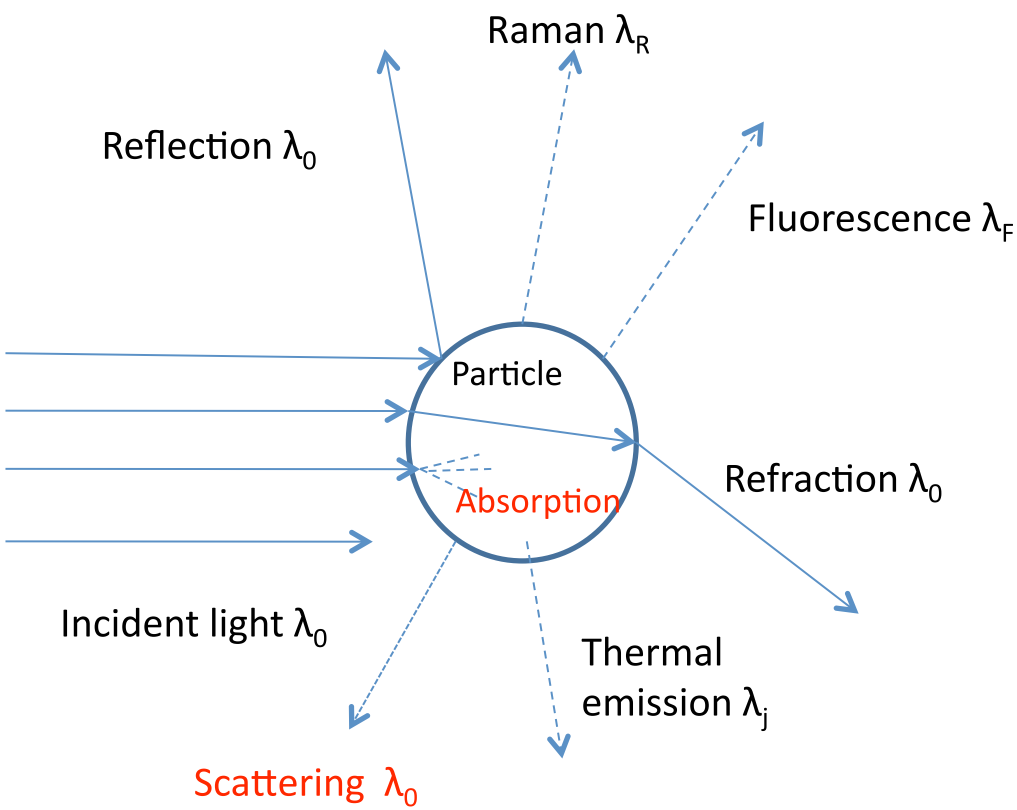
# Aerosol and Climate Change




Source: sciencedaily.com

- Aerosol can absorb or scatter sun light, warming effect and cooling effect
- The uncertainty of aerosol makes the total estimate so large
- Aerosol is the key to climate change

# Interaction between light and particle




- **Elastic scattering**

  The wavelength of the scattering is the same as that of the incident light

- **Inelastic scattering**

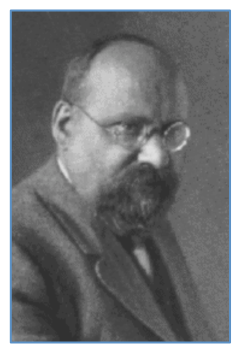
  The emitted light has a wavelength different from that of the incident light

For the interaction of solar radiation with atmospheric aerosols, elastic light scattering is the process of interest.

# Elastic Light Scattering

- Rayleigh Scattering
  – Spherical, small, non-absorbing,.
- Mie Scattering
  – Spherical, absorbing/non-absorbing, no size bound.

Lord Rayleigh

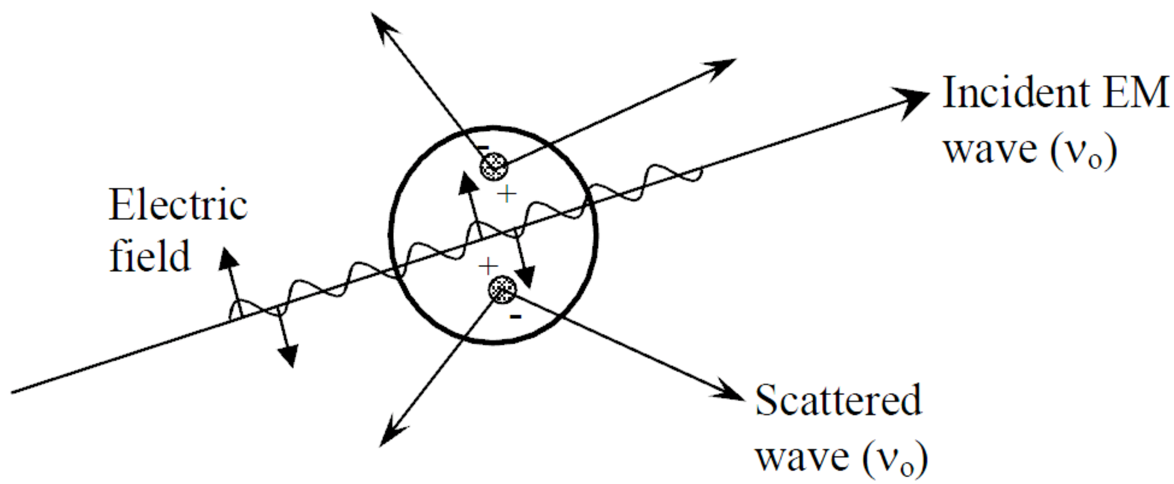


Gustav Mie

Source: Nobelprize.org

# Light Scattering

- Redirection of EM wave when encounters an obstacle or non-homogeneity (Scattering Aerosol).




Hahn, D. W. et al

- It is ***NOT*** a simple bounce of Photon.

# Light Scattering

## Three parameters that govern the scattering:

1. Wavelength λ of incident light
2. Size of the particle, *size parameter*

$$x = \frac{2\pi\ a}{\lambda}$$

3. the particle optical property relative to the surrounding medium, *refractive index*

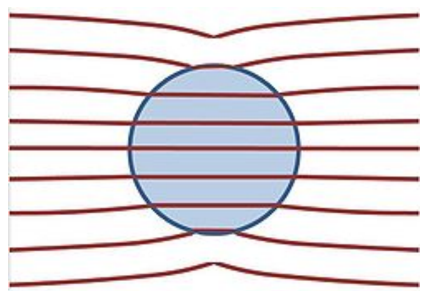
$$m = n - ik$$

Criteria for Rayleigh Regime x<<1, or |m| x<<1

# Rayleigh Theory

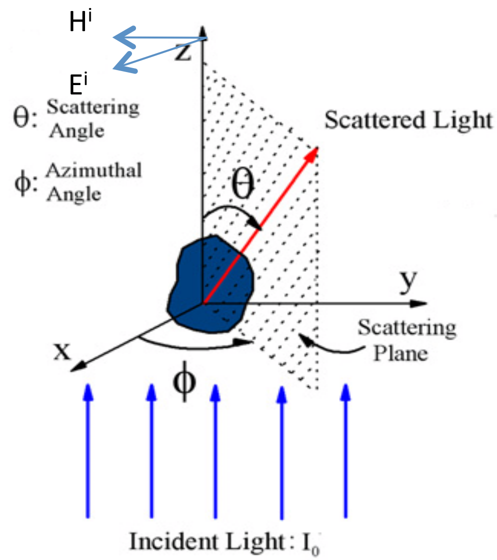
❖ **Rayleigh theory** describes the scattering of electromagnetic radiation by small spherical particles based on the dipole moment

❖ **Assumption:** particle is small compared to the wavelength so that the electric field inside the particle due to EM wave is uniform.



A uniform electric filed in side the particle

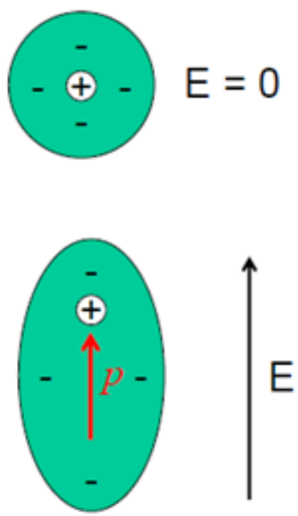
Source: wikimedia




Coordinate geometry for Rayleigh and Mie scattering,

Source: ipac.caltech.edu

# Rayleigh Theory




❖ Incident oscillation electric filed:
➔induced oscillation dipole moment

$$pe^{i\omega t} = 4\pi\varepsilon_2 a^3\left(\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+2\varepsilon_2}\right)E_0e^{i\omega t}$$

p: dipole moment a:particle radius
$E_0$: incident electric field ε:electric constant

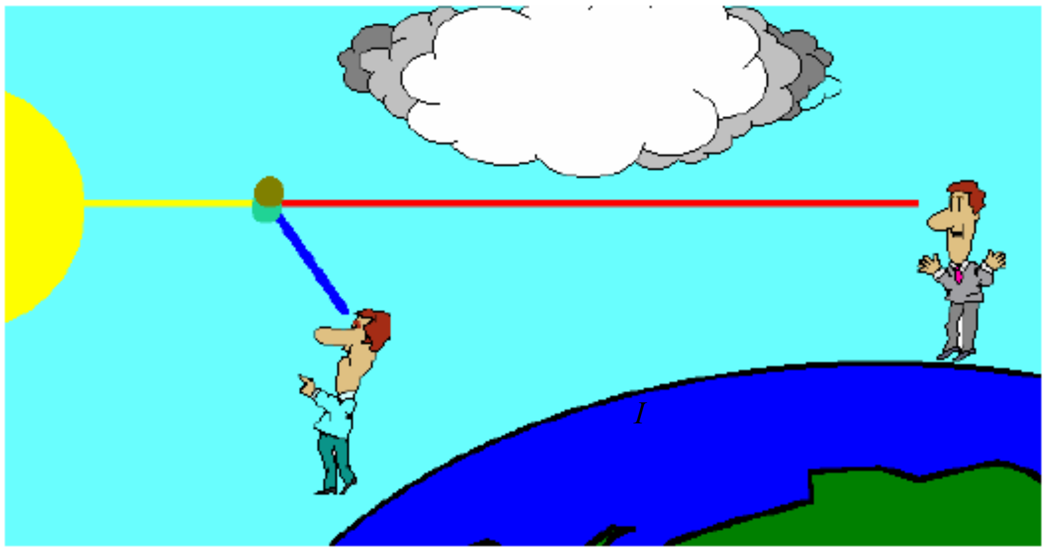
❖ Accelerating charges can emit radiate➔ the oscillation dipoles will radiate at the same frequency as the applied field in all the direction

$$I = \frac{8\pi^4a^6}{r^2\lambda^4}\left(\frac{n^2-1}{n^2+2}\right)^2\left(1+\cos^2\theta\right)$$

Blue sky

Source: M Kerker

# Blue sky & Red Sunsets



- Daytime sky looks blue on a clear day
- The sky looks red at sunrise and sun set

Why?

Blue light is scattered more efficiently than red

$$I_\lambda \propto \frac{1}{\lambda^4}$$

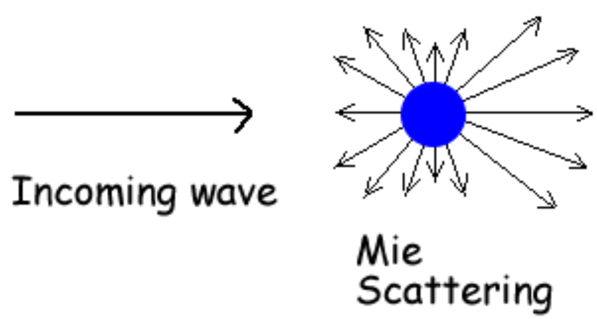
http://www.forbrf.lth.se/

# Mie scattering

❖ **Mie theory** describes the scattering and absorption of electromagnetic radiation by spherical particles through solving the Maxwell equations

❖ ***Key Assumptions***

i) Particle is a **sphere;**
ii) Particle is **homogeneous** **(therefore it is characterized by a single refractive index *m=n - ik at a given wavelength);***



# Solution of Mie scattering

❖ **Overview of Mie scattering solution for spheres:**

- Begin with Maxwell's equations
- Derive a wave equation in spherical polar coordinates
- Provide boundary conditions at surface of a sphere
- Solve the partial differential wave equation for dependence on r, θ, ϕ.

$$I_\phi = I_0 \frac{\lambda^2}{4\pi^2 r^2} i_1 \sin^2\phi,$$

$$I_\theta = I_0 \frac{\lambda^2}{4\pi^2 r^2} i_2 \cos^2\phi,$$

Intensities of scattering radiation

$$\sigma_{ext} = \frac{\lambda^2}{2\pi} \sum_0^n (2n+1) \mathrm{Re}\{a_n + b_n\}$$

$$\sigma_{scat} = \frac{\lambda^2}{2\pi} \sum_0^n (2n+1)\left(|a_n|^2 + |b_n|^2\right)$$

Cross sections

Fortunately, there are some people who enjoy solving these equations and setting up computer programs to calculate the results.

# Mie Scattering Code

**Code for Mie scattering by a single sphere**

| Year | Name | Authors | Language | Short description |
|---|---|---|---|---|
| 1983 | BHMIE | Craig F. Bohren | Matlab, Fortran | Mie solution by a homogeneous sphere |
| 2002 | MiePlot | Philip Laven | VB | Solution for a number of wavelength |
| 2003 | Mie single | Gareth Thomas | IDL | Solution for both single and populations of particle with log-normal PSD |

**Codes for electromagnetic scattering by a layered sphere**

| Year | Name | Authors | Language | Short description |
|---|---|---|---|---|
| 1993 | IFCS | Thomas Kaiser | Fortran | Compute the scattered filed of sphere with 1 to 2 coatings |
| 2004 | | M.Jonasz | Fortran | Compute parameters of single coated sphere |
| 2003 | | L. Liu | C | Light scattering by a coated sphere |

Source: Wikipedia.org

# Calculate optical parameters

For a single spherical particle, the Mie theory gives the extinction , scattering and absorption cross-sections, the scattering amplitudes and phase matrix.

Particle size
Refractive index, m (λ)

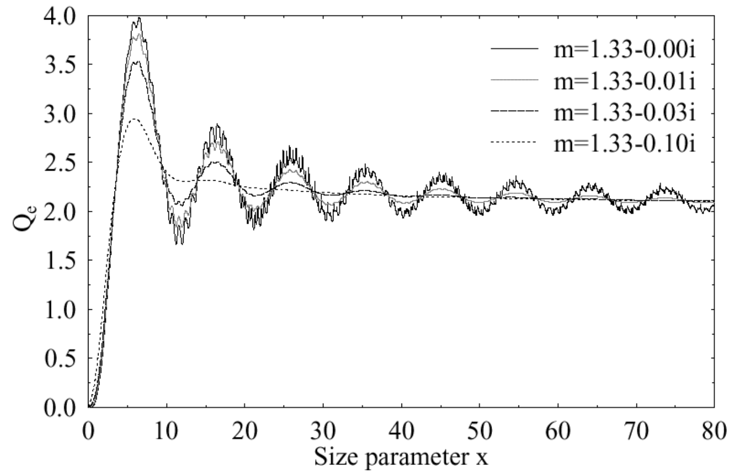
↓ **Mie theory**

Scattering cross section, σs
Absorption cross section, σa
Extinction cross section, σe
(as a function of a particle size and wavelength)

↓ **Mie theory**

Scattering coefficient, σs
Absorption coefficient, σa
Extinction coefficient, σe
(as a function of a particle size and wavelength)

# Mie scattering



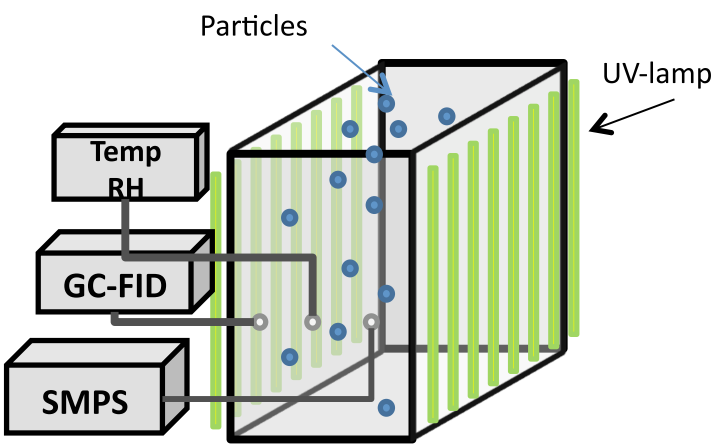

**Some highlights of Mie scattering results:**

Extinction efficiency vs size parameter (no absorption):

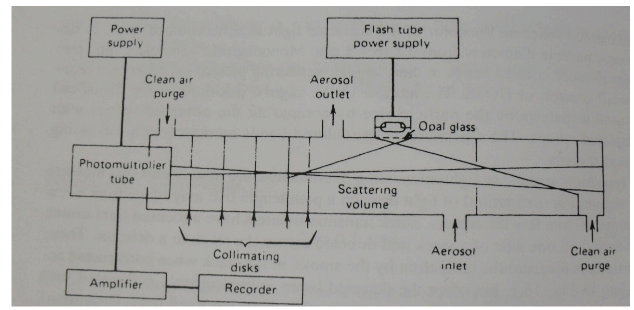
1) Small in Rayleigh limit Qext $x \propto x^4$
2) Largest $Q_{ext}$ when particle and wavelength have similar size.
3) Qext $\rightarrow$ 2 in geometric limit ($x \rightarrow \infty$).
4) Oscillations from interference of transmitted and diffracted waves.
5) Period in x of interference oscillations depends on the refractive index.

Absorption reduces interference oscillations and kills ripple structure.

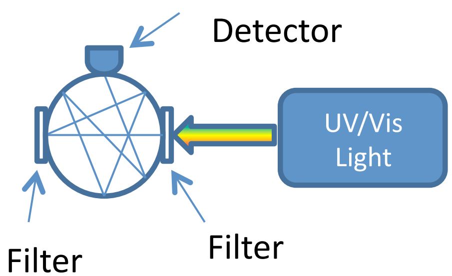
# Apply Mie Scattering to aerosol study




1) Teflon Chamber




3) Nephelometer




2) UV/Vis spectrometer

1. Chamber experiment → particle size, concentration
2. UV/Vis →absorption coefficient,
3. Nephelometer → extiction coefficient, 3 wave length
4. Input the above data into Mie scattering code and find out the refractive index (m)
5. Compare the computed (m) to the measured m
   m can be measured by the electron energy-loss spectroscopy using TEM equipped with a Gatan TriDiem spectrometer (Duncan T. L. Alexander, *et al. Science 321, 833,2008 )*

# References

1. M. Kerker. The scattering of light and other electromagnetic radiation. Academic, New York. 1969.
2. H.C. van de Hulst. Light scattering by small particles. John Wiley & Sons, New York, 1957.
3. C.F. Bohren and D.R. Huffman. Absorption and scattering of light by small particles. John Wiley & Sons, New York, 1983.
4. http://plaza.ufl.edu/dwhahn/Diagnostic%20Tutorials.html
5.Willian C. Hinds. Aerosol Technology, John Wiley & Sons, New York, 1998.
6. John H. Seinfeld and Spyros N. Pandis. Atmospheric Chemistry and Physics. John Wiley & Sons, New York, 1997.
7. Benjamin Chu, Laser light scattering, Academic, New York, 1974.

# Thank you !

# What can the code do

**Interactive Mie Scattering Calculator at** http://omlc.ogi.edu/calc/mie_calc.html

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## Mie Scattering Calculation

by Scott Prahl

These values were calculated using the interactive Mie Scattering Calculator at http://omlc.ogi.edu/calc/mie_calc.html

### Input Parameters

| | | |
|---|---|---|
| Sphere Diameter | 1.0 | microns |
| Refractive Index of Medium | 1.0 | |
| Real Refractive Index of Sphere | 1.5 | |
| Imaginary Refractive Index of Sphere | 0 | |
| Wavelength in Vacuum | 0.6328 | microns |
| Concentration | 0.1 | spheres/micron$^3$ |

### Calculated results

| | | |
|---|---|---|
| Wavelength in Medium | 0.6328 | microns |
| Size Parameter | 4.9646 | |
| Average Cosine of Phase Function | 0.70765 | |
| Scattering Efficiency | 3.8962 | |
| Extinction Efficiency | 3.8962 | |
| Backscattering Efficiency | 1.9428 | |
| Scattering Cross Section | 3.0601 | micron$^2$ |
| Extinction Cross Section | 3.0601 | micron$^2$ |
| Backscattering Cross Section | 1.5259 | micron$^2$ |
| Scattering Coefficient | 306 | mm$^{-1}$ |
| Total Attenuation Coefficient | 306 | mm$^{-1}$ |